Envelope Model for the Propagation of an Intense Proton Beam Through Thin Foils*

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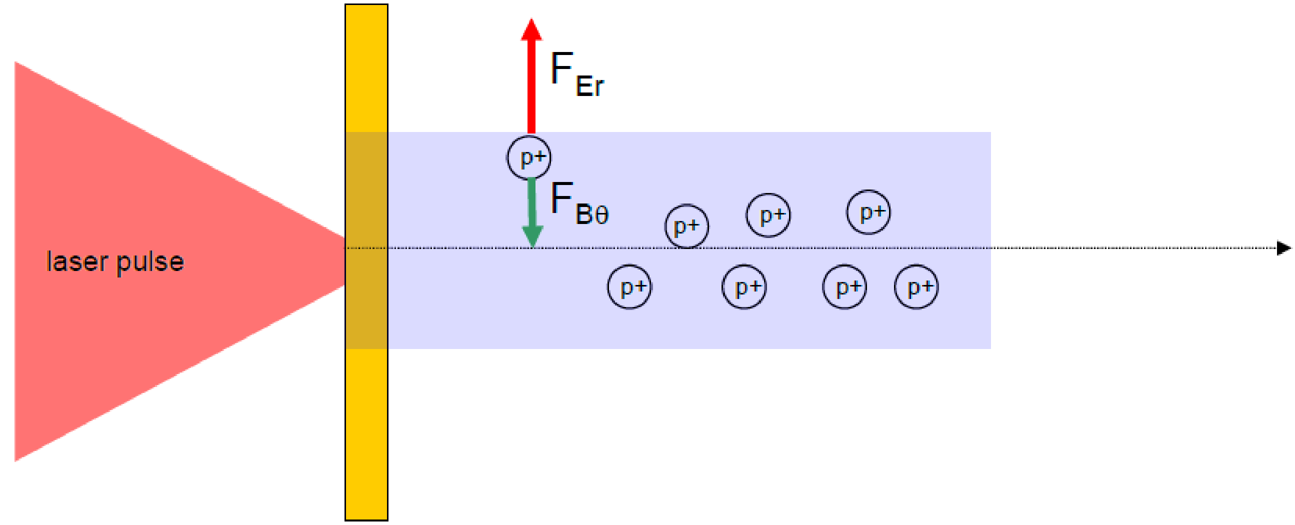
Indirect production of Japan-US program exchanges Chiharu and Joe in Saitama, Japan 10/05/2011

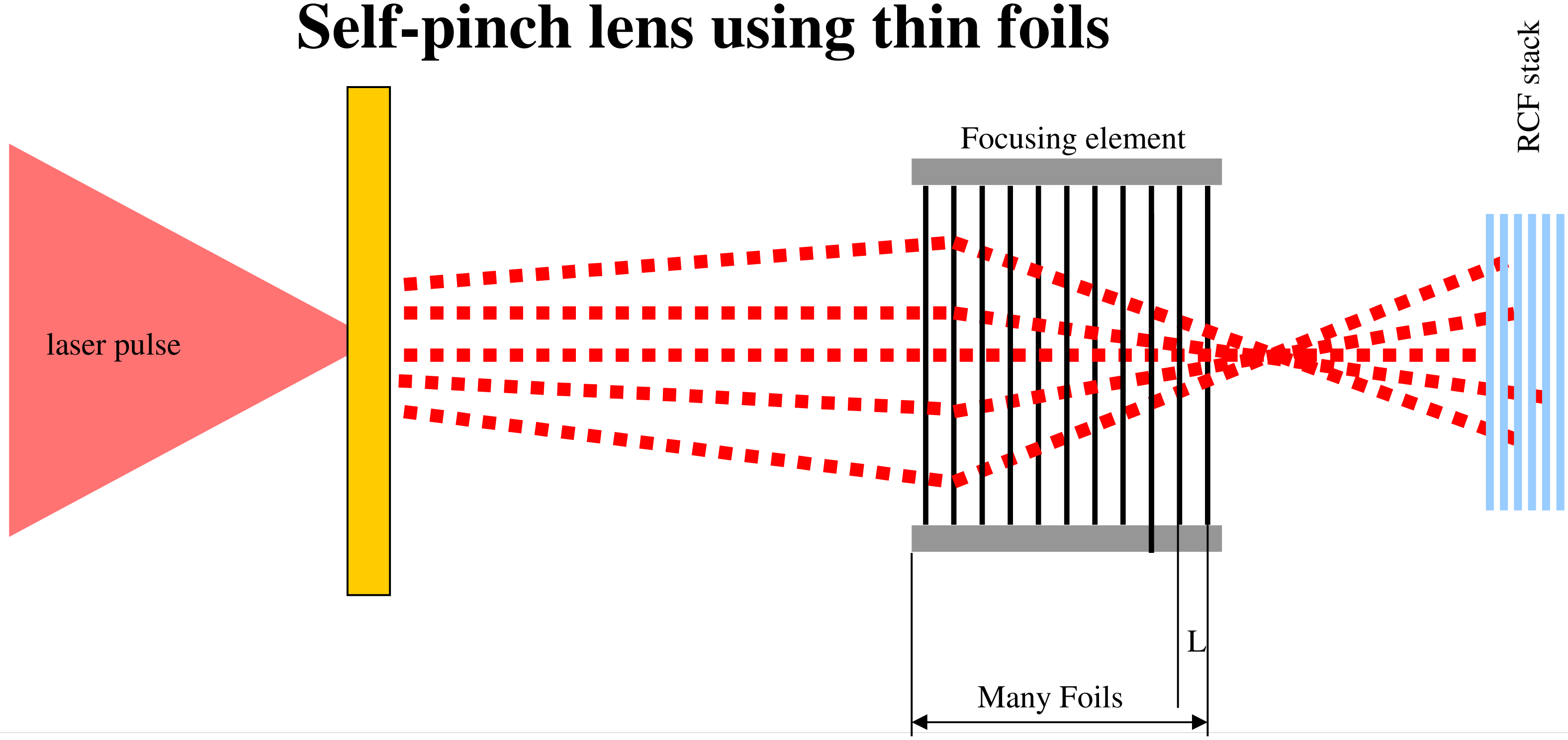
Lund, Kikuchi, and Davidson, Generation of initial kinetic distributions for simulation of long-pulse charged particle beams with high space-charge intensity, Phys. Rev. Special Topics – Accelerators and Beams 12, 114801 (2009)

Concept: G. Logan

Protons accelerated by plasma sheath on back of laser illuminated foil



Self-pinch lens using thin foils



Outline

1. Geometry and Beam Model
   - Uniform Density Beam
   - Gaussian Density Beam

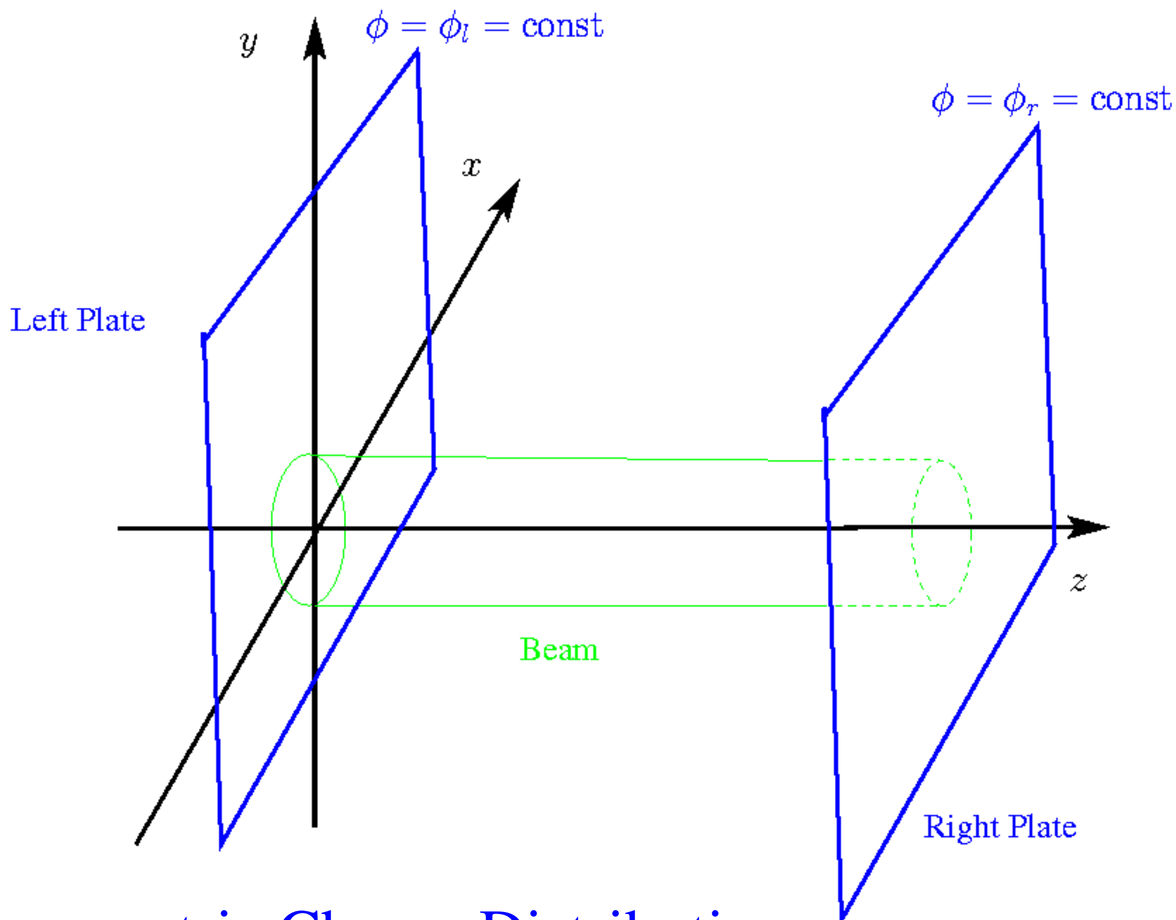
2. Overview of Steps to Derive Envelope Model
   - Electrostatic Self-Field
   - Magnetostatic Self-Field
   - Particle Trajectory Equations
   - Statistical Average of Particle Equations

3. Transverse Envelope Model
   - Envelope Equations
   - Example Transport Solution – GSI Experiment

4. Conclusions

Geometry

Intense beam filling region between two closely spaced, thin conducting foil planes:



Axisymmetric Charge Distribution:

$$\rho(\mathbf{x}) = \rho(r) \qquad r = \sqrt{x^2 + y^2}$$

Axial Current Distribution:

$$\mathbf{J}(\mathbf{x}) \simeq \rho(r) V_z \hat{\mathbf{z}} \qquad V_z = \langle v_z \rangle_\perp$$

Charge Density

Examine two forms of beam charge density:

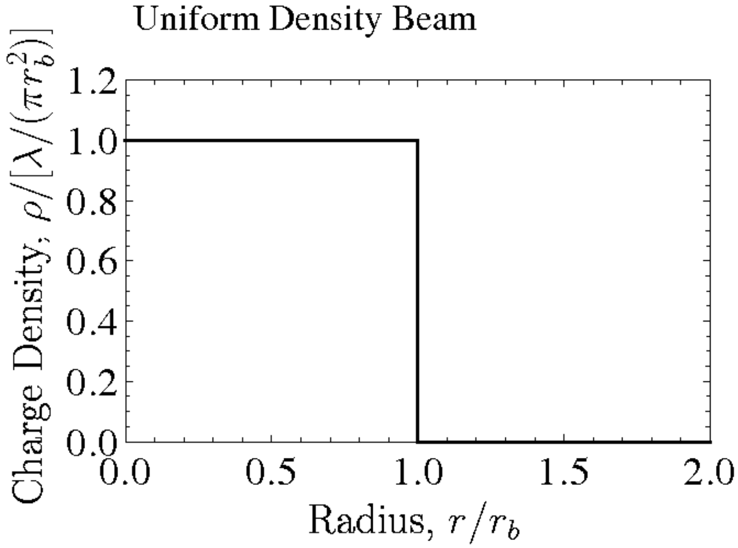
1/ Uniform Density Beam

- Appropriate to model a space-charge dominated beam in linear focus field

$$\rho(r) = \begin{cases} \frac{\lambda}{\pi r_b^2}, & 0 \leq r \leq r_b \\ 0, & r_b \leq r \end{cases}$$

$\lambda = \text{const}$ Line Charge
$r_b = \text{const}$ Beam Radius

$$r_b = 2\langle x^2 \rangle_\perp^{1/2}$$



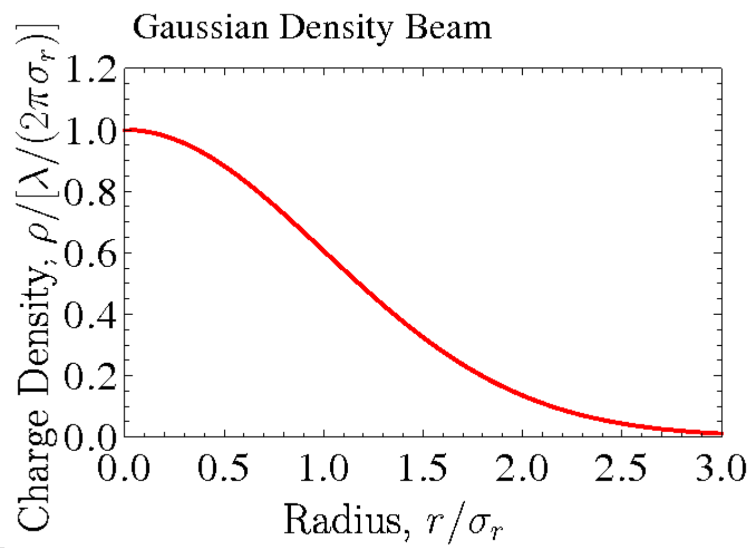
2/ Gaussian Density Beam

- Appropriate to model a beam without focusing

$$\rho(r) = \frac{\lambda}{2\pi\sigma_r^2} e^{\frac{r^2}{2\sigma_r^2}}$$

$\lambda = \text{const}$ Line Charge
$\sigma_r = \text{const}$ rms “sigma”

$$\sigma_r = \langle x^2 \rangle_\perp^{1/2}$$



Model Derived

Self-Fields:

- Magnetic self-field: magnetostatic free-space form
- Electric self-field: electrostatic solved with Green's Function in presence of conducting foils

Particle Equations of Motion:

- Derived with self-fields and linear applied solenoid focusing field

Statistical Envelope Equations:

- Derived taking averages over assumed beam distributions
- Rapid z-variation of electric self-field averaged between foils
- Carried out for both Uniform and Gaussian charge distributions

Envelope Equation for a Gaussian Density Beam

Outlined procedure gives:

$$\frac{d^2}{dz^2}\sigma_r + \kappa(z)\sigma_r + \frac{\gamma_b^2}{4}\left[\beta_b^2 - F_g\right]\frac{Q}{\sigma_r} - \frac{\varepsilon_{x,\mathrm{rms}}^2}{\sigma_r^3} = 0$$

rms beam size:

$$\sigma_r = \langle x^2 \rangle_\perp^{1/2}$$

rms edge emittance:

$$\varepsilon_{x,\mathrm{rms}} \equiv 4\left[\langle x^2\rangle_\perp \langle x'^2\rangle_\perp - \langle x x'\rangle_\perp^2\right]^{1/2}$$

Applied magnetic solenoid focusing function:

$$\kappa(z) = \frac{qB_z}{2m\gamma_b\beta_b c}$$

Dimensionless perveance:

$$Q \equiv \frac{q\lambda}{2\pi\epsilon_0 m\gamma_b^3\beta_b^2 c^2} = \frac{qI}{2\pi\epsilon_0 m\gamma_b^3\beta_b^3 c^3}$$

Dimensionless form factor:

$$F_g \equiv 4(\sigma_r/L)^2 \int_0^\infty dk\, \frac{1 - \cosh(k) + \frac{k}{2}\sinh(k)}{\sinh(k)} e^{-k^2(\sigma_r/L)^2}$$

Function only of beam "aspect ratio" $\dfrac{\perp \text{ Beam Size}}{\text{Foil Spacing}} = \dfrac{\sigma_r}{L}$

"Unfamiliar" terms in the envelope equation:

$$\frac{(\gamma_b \beta_b)^2}{4} \frac{Q}{\sigma_r}$$

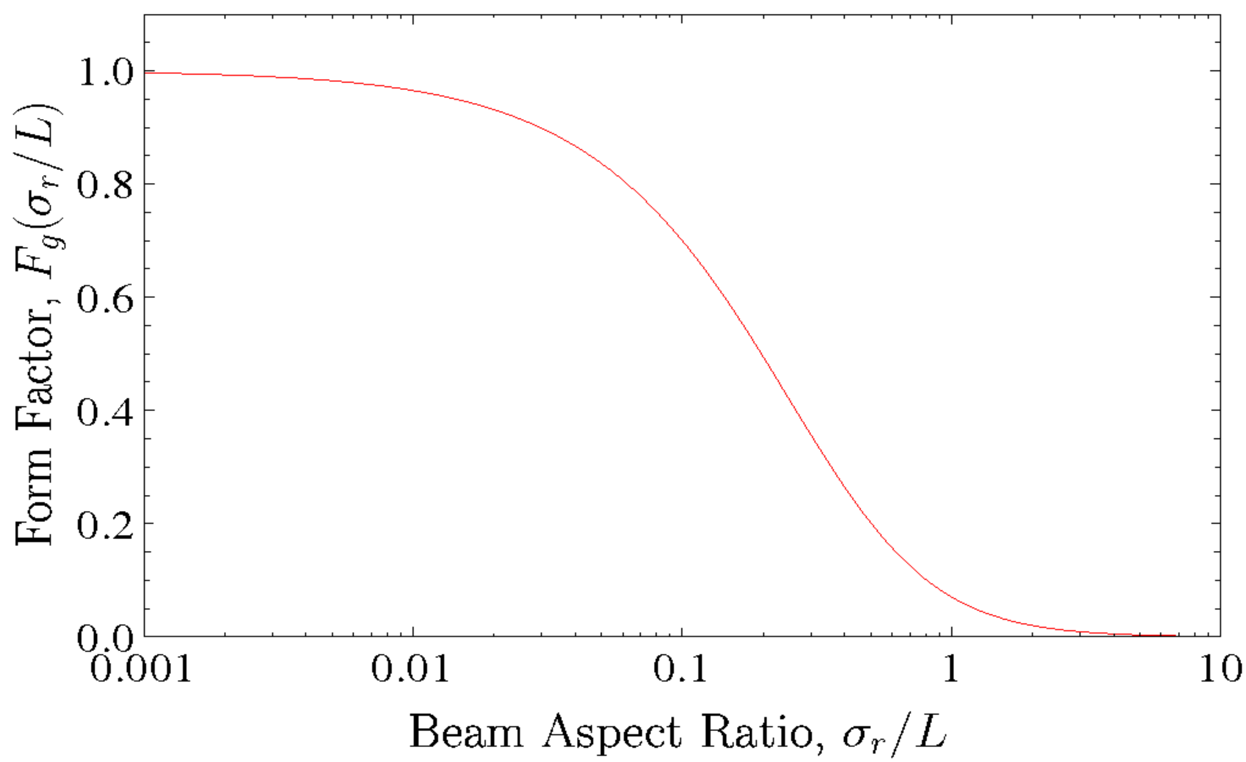
Magnetic focusing from beam current

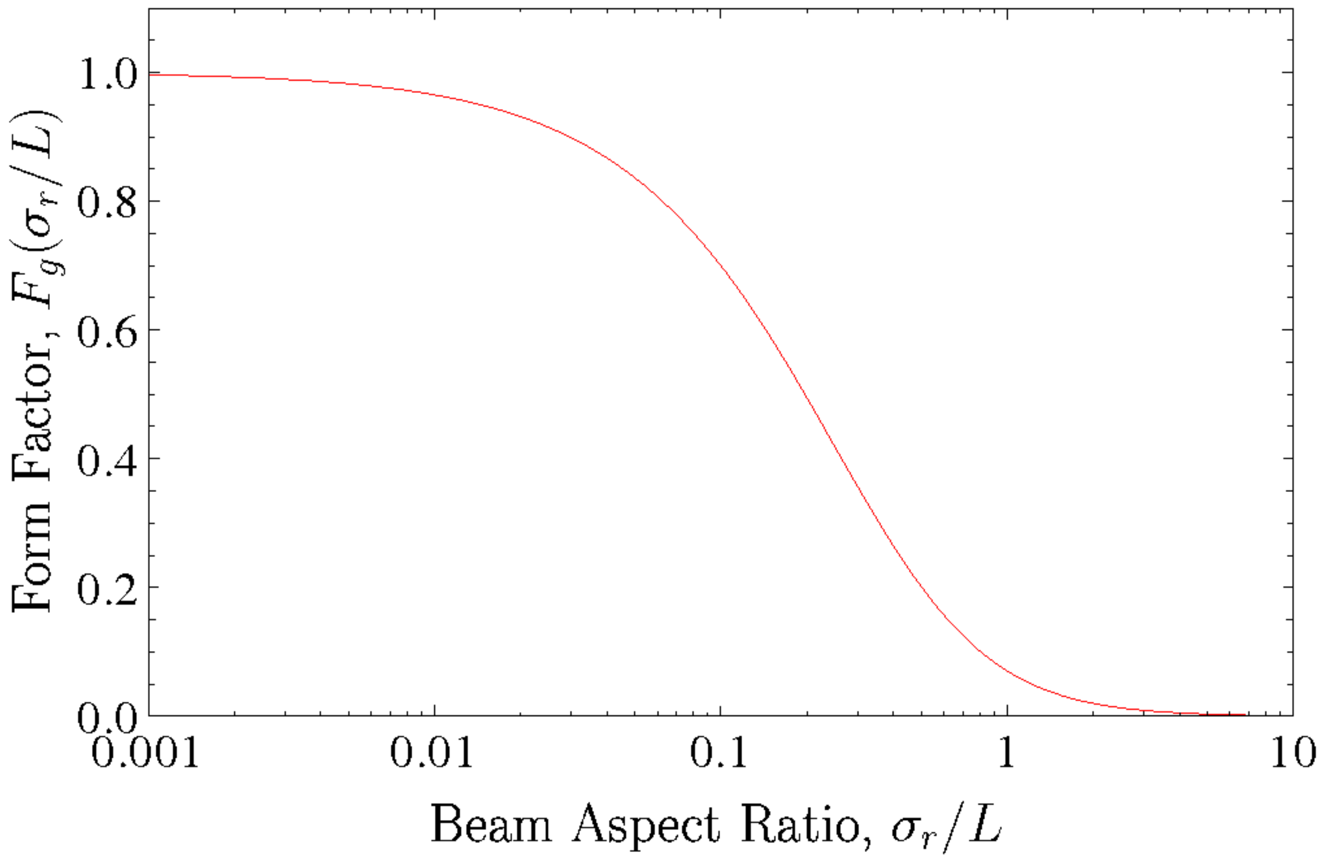
$$-\frac{\gamma_b^2}{4} F_g \frac{Q}{\sigma_r}$$

Electric de-focusing from beam (nonlinear) beam space-charge

Form factor, Gaussian charge density beam

- Effectively gives the attenuation of defocusing radial electric field in the presence of foils relative to vacuum (no foils) value





Small aspect ratio beams, the usual envelope equation of a beam in vacuum is recovered:

$$\lim_{\sigma_r/L\to 0} F_g = 1 \quad \Longrightarrow \quad \frac{d^2}{dz^2}\sigma_r + \kappa(z)\sigma_r - \frac{Q}{4\sigma_r} - \frac{\varepsilon_{x,\mathrm{rms}}^2}{\sigma_r^3} = 0$$

Large aspect ratio beams, electric de-focusing is negligible:

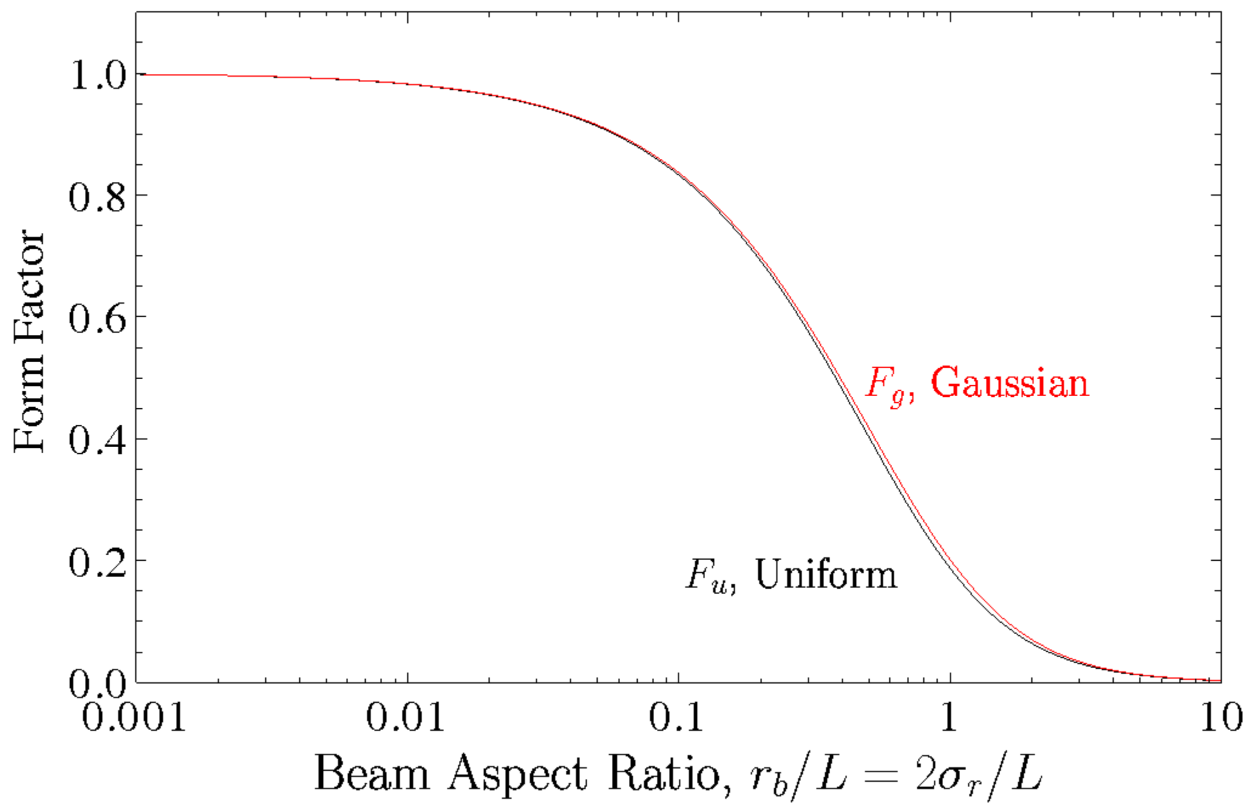
$$\lim_{\sigma_r/L\to \infty} F_g = 0 \quad \Longrightarrow \quad \frac{d^2}{dz^2}\sigma_r + \kappa(z)\sigma_r + \frac{(\gamma_b\beta_b)^2}{4}\frac{Q}{\sigma_r} - \frac{\varepsilon_{x,\mathrm{rms}}^2}{\sigma_r^3} = 0$$

Differences in envelope solutions between uniform and Gaussian charge density models expected to be small

For equivalent beam sizes $r_b = 2\langle x^2 \rangle_\perp^{1/2} = 2\sigma_r$

and the *same* parameters, all terms in the uniform and Gaussian charge density envelope models are the same other than the form factors $F_u, \; F_g$.

Compare form factors for equivalent aspect ratios: $\dfrac{r_b}{L} = 2\dfrac{\sigma_r}{L}$

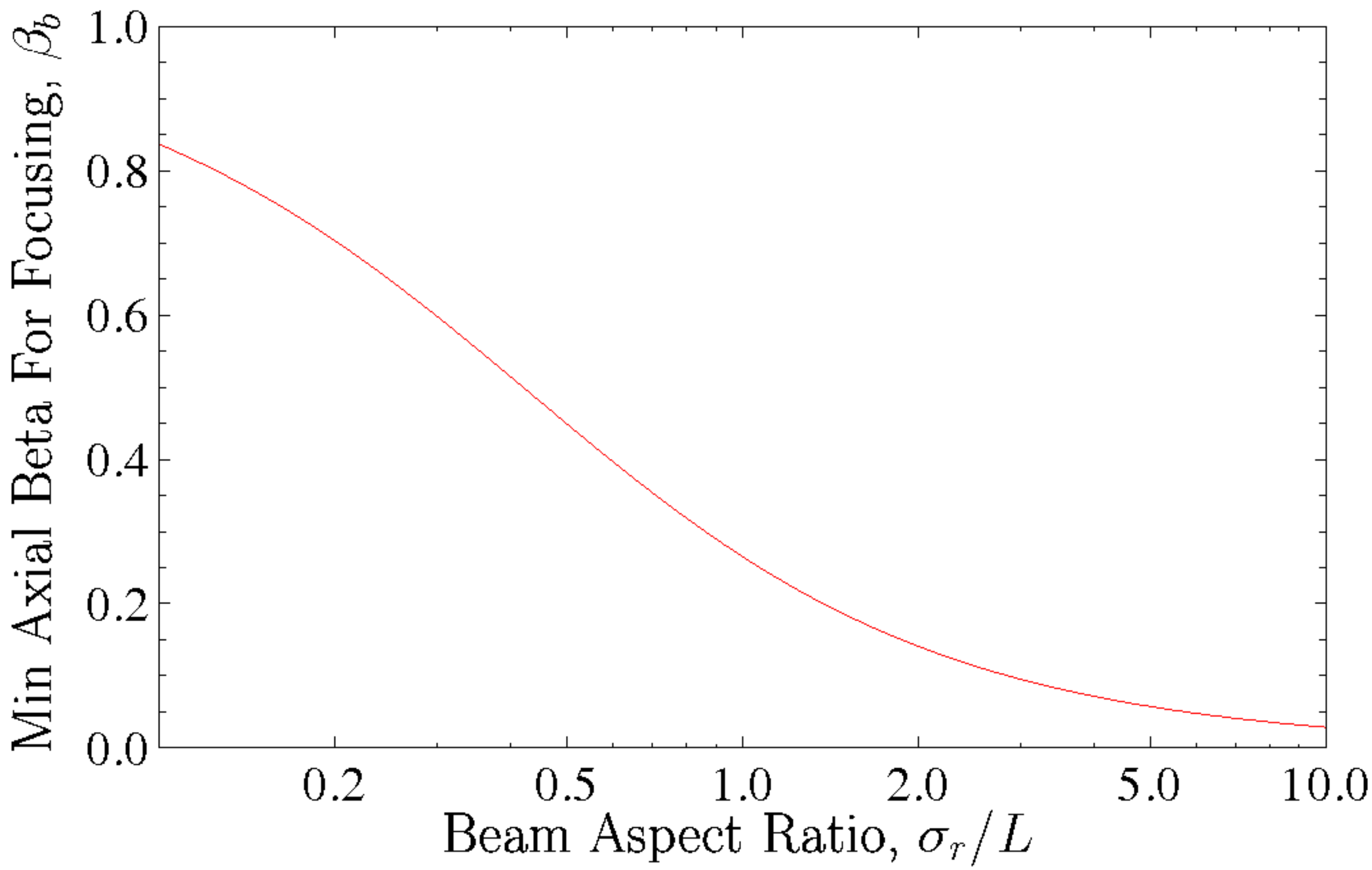


- Little difference for equivalent uniform and Gaussian beams

Beam aspect ratio needed for magnetic focus force to be larger than electric defocusing force

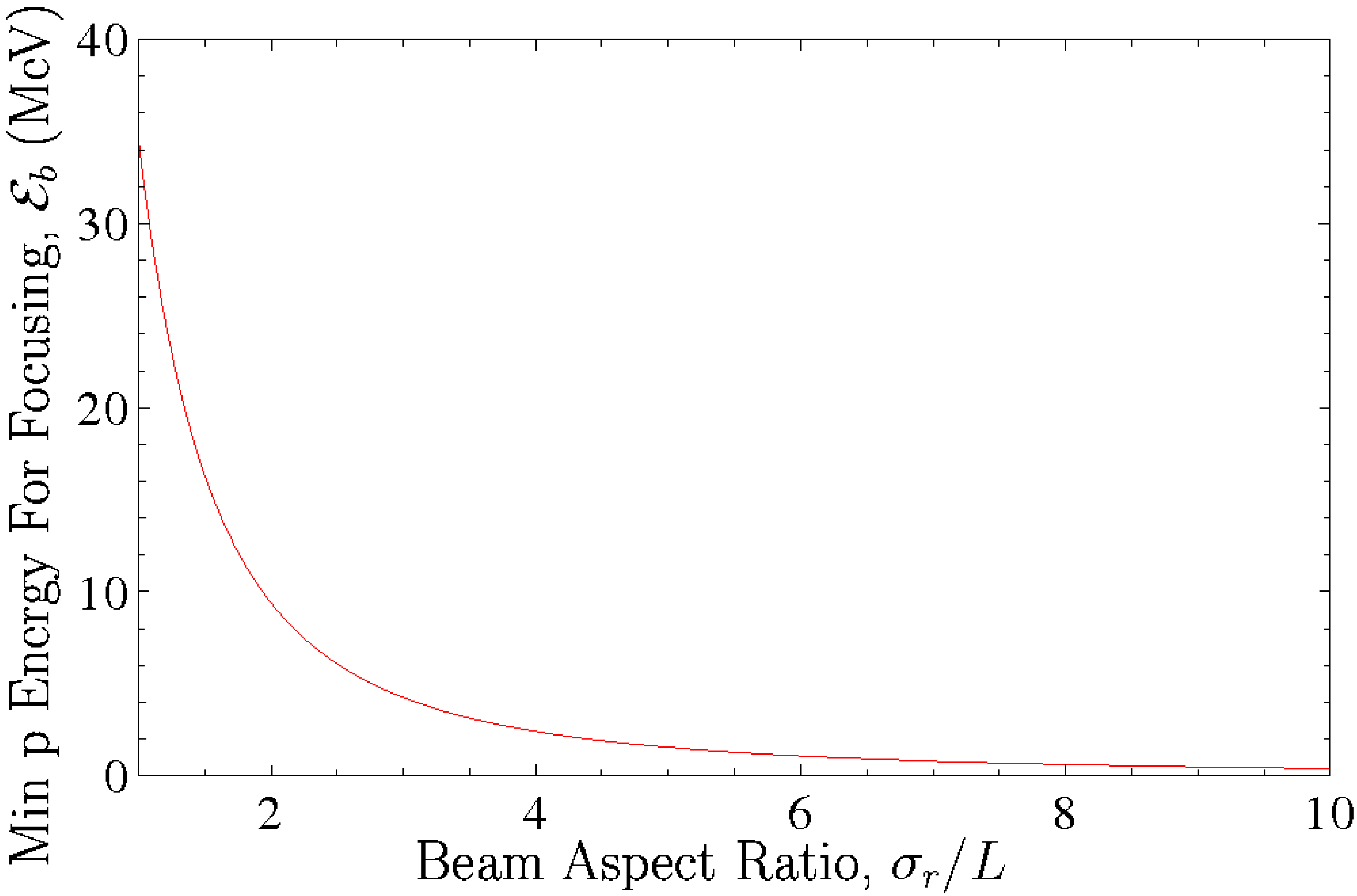
$\beta_b > \sqrt{F_g}$ $\Longleftrightarrow$ Net Focusing

Magnetic focus Force larger than electric defocus force



Express result for protons over a range likely relevant to laser produced proton beam experiments

$$\mathcal{E}_b = mc^2(\gamma_b - 1) \qquad \gamma_b = \frac{1}{\sqrt{1-\beta_b^2}}$$



Example Transport Solution – GSI Experiment

Apply envelope formulation to estimate the max acceptable foil spacing to focus the GSI laser produced proton beam using the PHELIX laser in experiments to be carried out by M. Roth's group at GSI/TU-Darmstadt

Proton Beam Parameters:

$$p^+ \quad \mathcal{E}_b = 10\ \text{MeV} \quad \Longleftrightarrow \quad \beta_b = 0.145 \quad \gamma_b = 1.011$$

$$I = 400.5\ \text{A} \quad \Longleftrightarrow \quad \sim 10^{10}\ p^+\ \text{in 4 ps}$$

$$\Longrightarrow \quad Q = \frac{q\lambda}{2\pi\epsilon_0 m \gamma_b^3 \beta_b^2 c^2} = 8.13 \times 10^{-3}$$ Perveance

$\varepsilon_{x,\text{rms}} \simeq 0$ Emittance small ... take zero to simply illustrate trends

$\sigma_r \sim 200\ \mu\text{m}$ Initial rms size of beam at entry to foil lens

$\sigma_r' \sim 393\ \text{mrad}$ Initial rms angle of beam at entry to foil lens

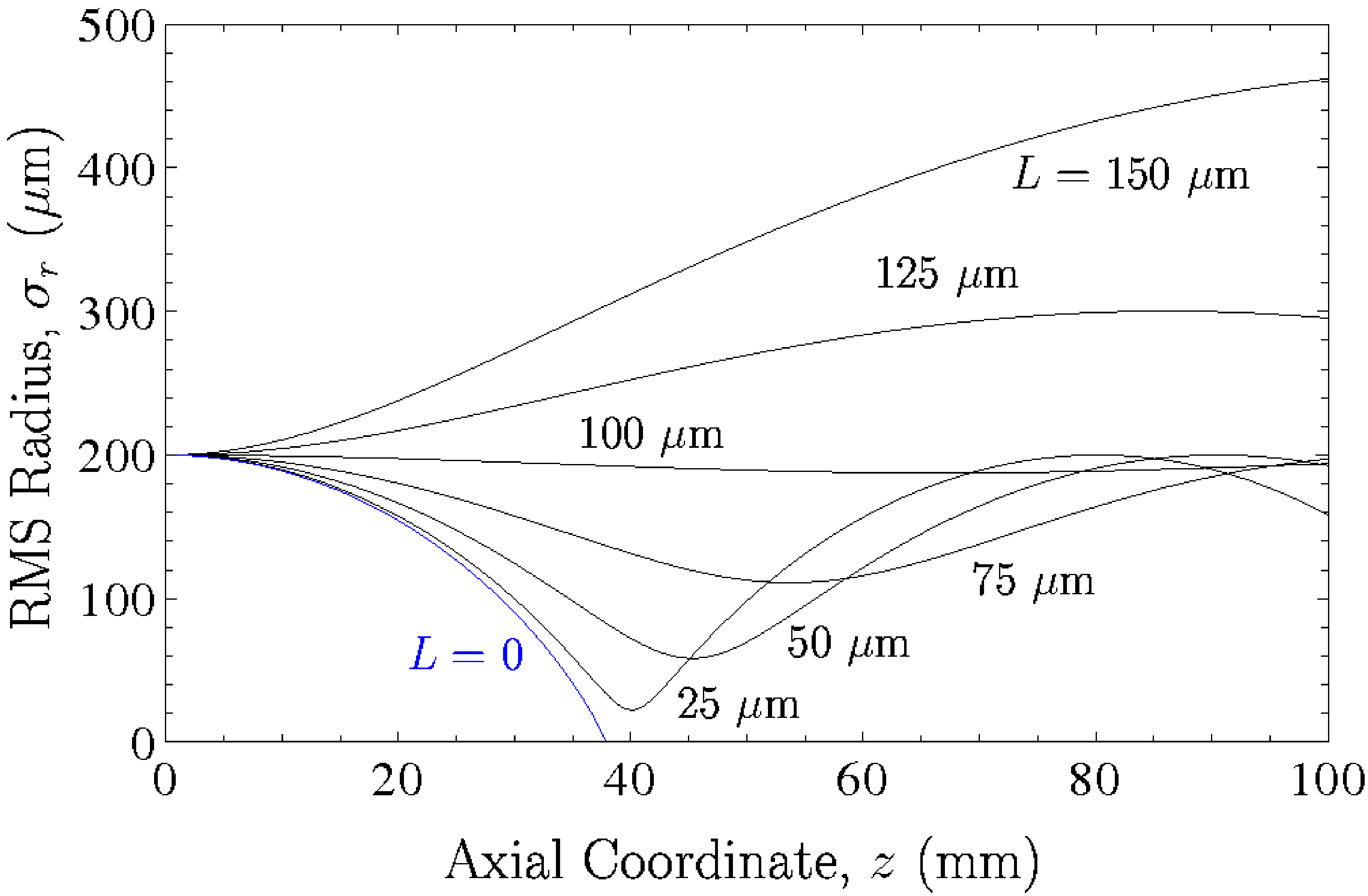
$\sim 22.5^\circ$

Apply Gaussian beam model with a variety of foil separations L

- Not much difference expected compared to uniform beam model

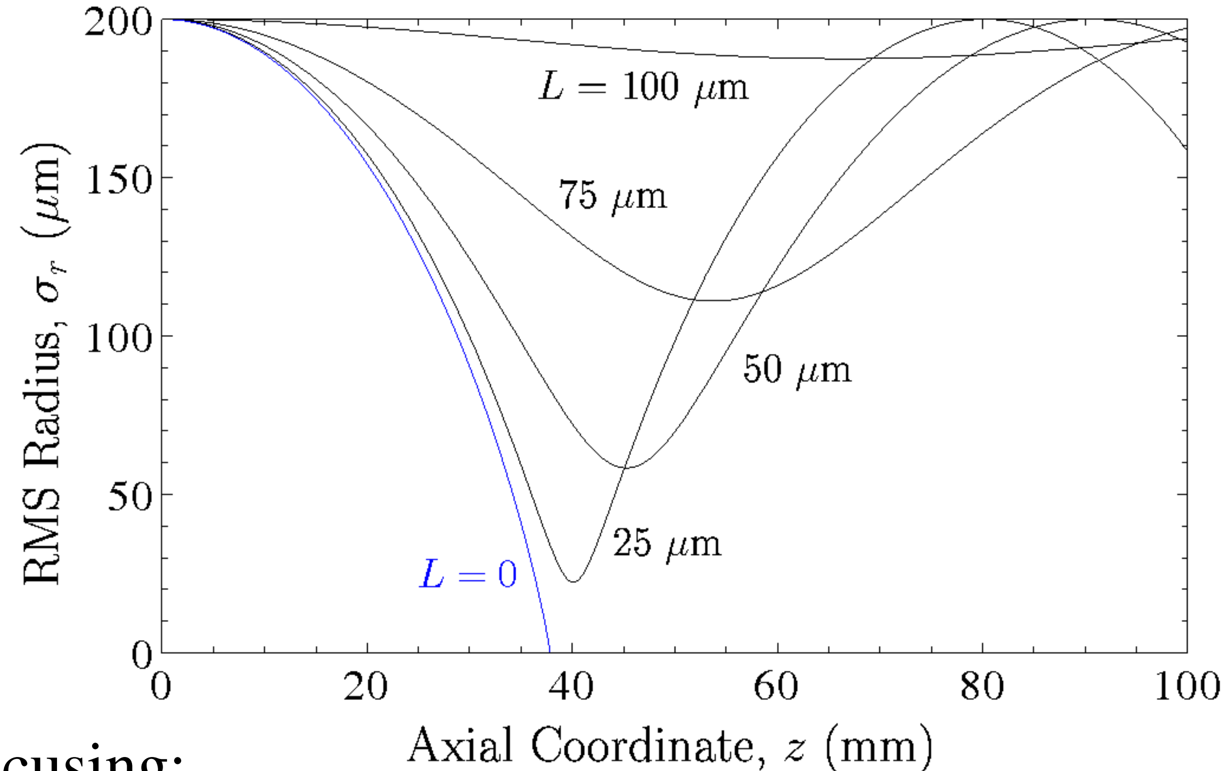
Envelopes for zero initial divergence with varied foil spacing

Initial Envelope: $\sigma_r = 200\ \mu\text{m} \quad \sigma_r' = 0$



- Pinch focus spot achieved for plate spacing less than 100 microns

Expand Scale: For foil spacings leading to net focusing



For net focusing:

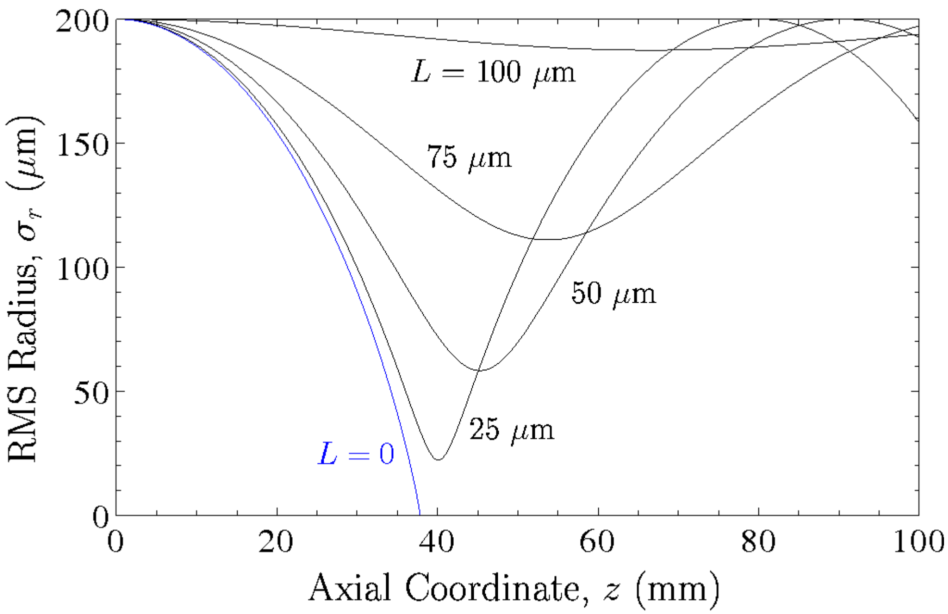
$$\sigma_r/L > 1.94 \quad \Longleftrightarrow \quad \begin{aligned} L &= 100\ \mu\text{m}: & \sigma_r &= 194\ \mu\text{m} \\ L &= 75\ \mu\text{m}: & \sigma_r &= 145\ \mu\text{m} \\ L &= 50\ \mu\text{m}: & \sigma_r &= 96.8\ \mu\text{m} \\ L &= 25\ \mu\text{m}: & \sigma_r &= 48.4\ \mu\text{m} \end{aligned}$$

- Compression beyond min aspect ratio due to "inertia" of converging beam
- Final stage of compression faster when beam becomes small
  - Eventually aspect ratio poor and space-charge term repels resulting in min spot for $L \neq 0$

Even modest initial beam divergence likely precludes concept

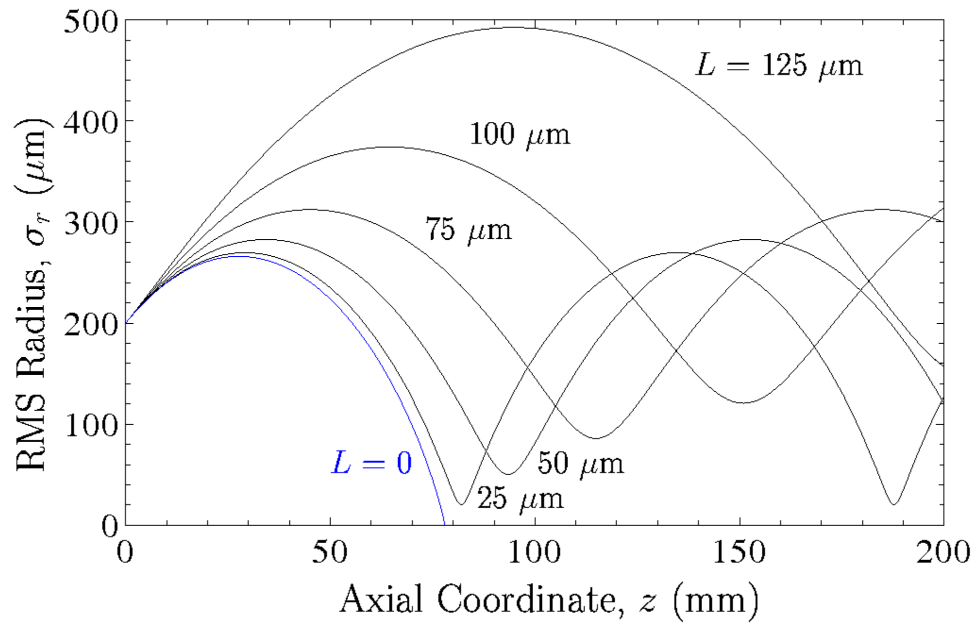
Reference Initial Envelope:

$$\sigma_r = 200\ \mu\text{m} \quad \sigma'_r = 0$$



Diverging Initial Envelope:

$$\sigma_r = 200\ \mu\text{m} \quad \sigma'_r = 5\ \text{mrad}$$



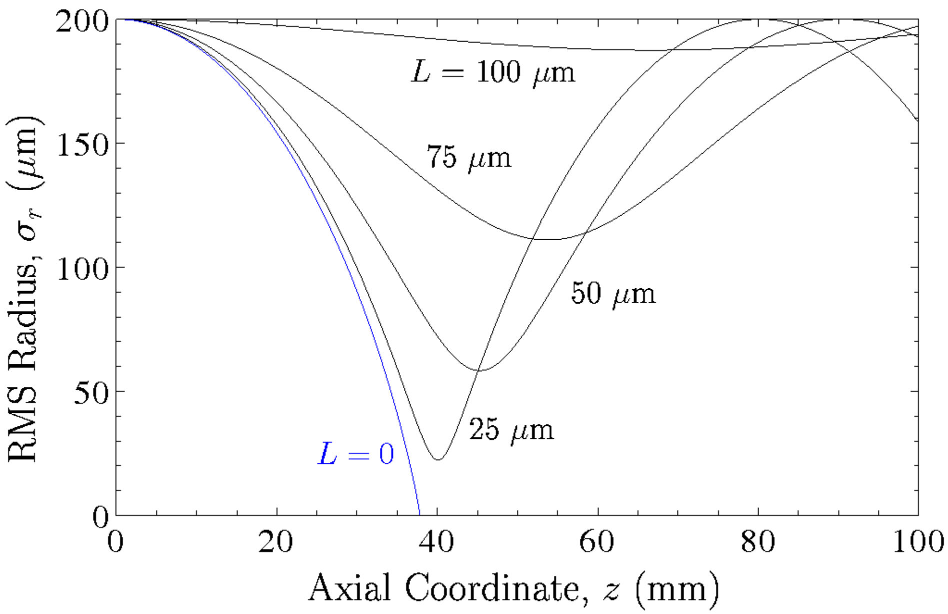
Modest envelope divergence results in much longer propagation distance to spot -- large divergence of planned GSI experiment is untenable

- Magnetic focusing weak for larger beam resulting in long length to spot
  - Aspect ratio improves but divergence increases length to overcome initial weak focusing of large beam
- Final spot size moderately reduced in cases where focusing weak
  - Inertia of compression larger when beam finally bent toward axis

Even modest initial beam convenance aids considerably

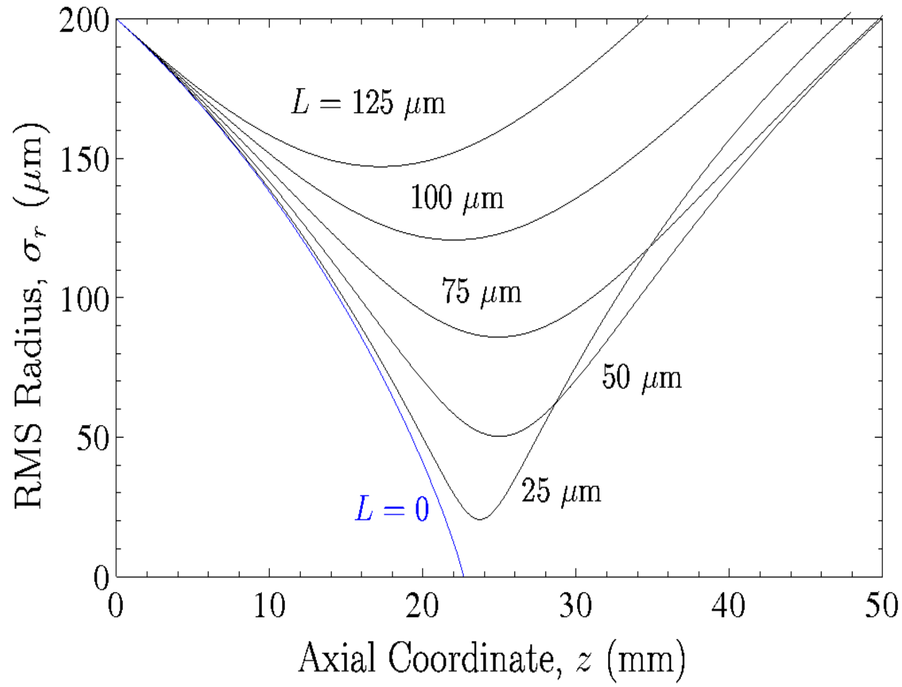
Reference Initial Envelope:

$$\sigma_r = 200\ \mu\text{m} \quad \sigma_r' = 0$$



Converging Initial Envelope:

$$\sigma_r = 200\ \mu\text{m} \quad \sigma_r' = -5\ \text{mrad}$$



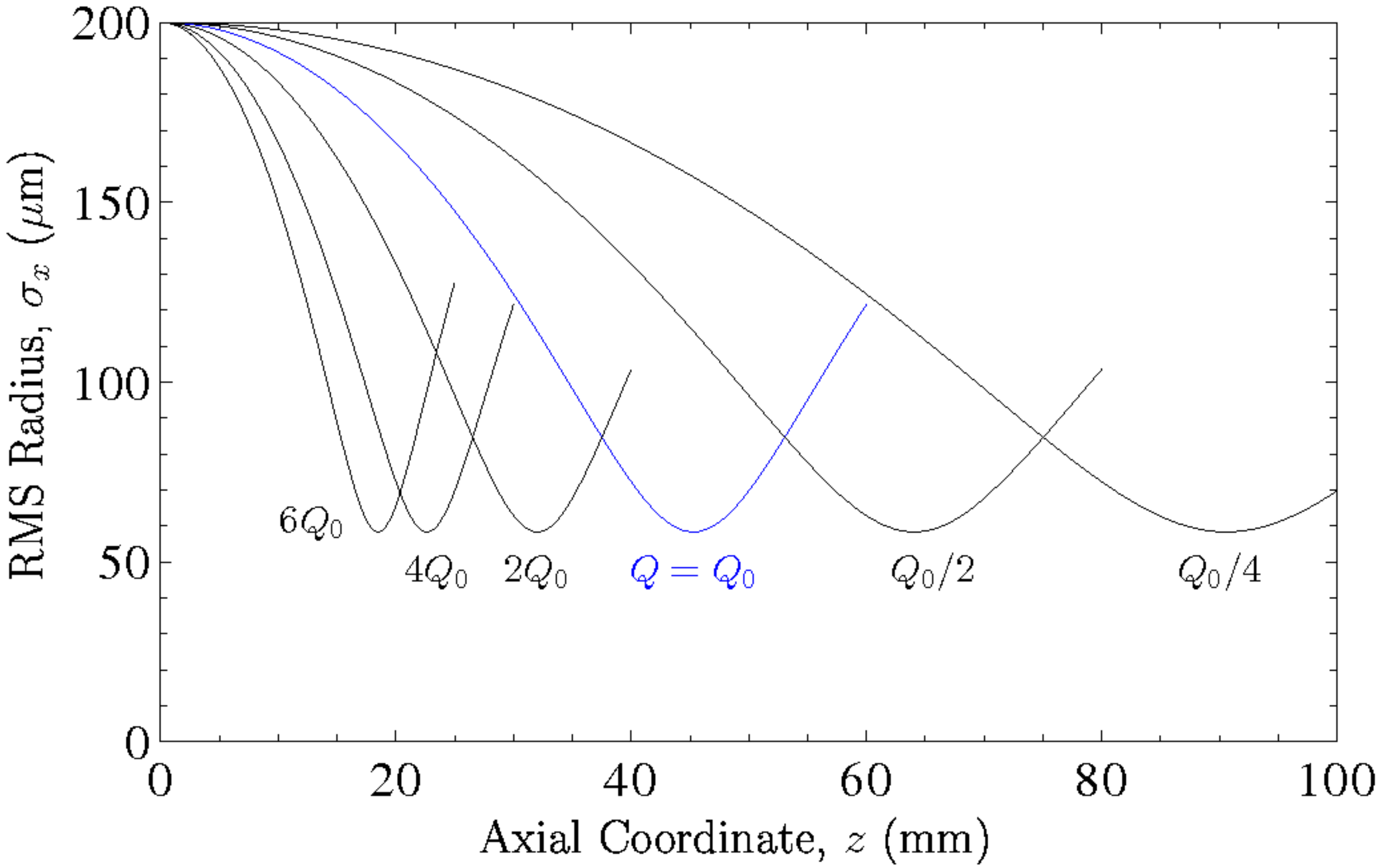
Envelope convergence helps speed slow part of initial compression

- Magnetic focusing weak for larger beam resulting in long length to spot
  - Aspect ratio improves but divergence increases length to overcome initial weak focusing of large beam
- Final spot size improves a little
  - Inertia of compression slightly increased when beam finally bent to axis

Higher beam perveance Q increases strength of focusing

Examine L = 50 micron foil spacing case as perveance varied:

$$Q = Q_0 = 8.13 \times 10^{-3} = \text{Base Case} \qquad Q = \frac{qI}{2\pi\epsilon_0 m\gamma_b^3\beta_b^3c^3}$$



Beta determines focus/defocus but larger Q increases strength of effect

- Higher currents focus better
  - Less distance/foils to peak compression

Summary guidance suggested by envelope model

- Foil spacing ~ 50 microns or less to be confident of seeing effect
  - Consistent with fabrication limits
  - Larger separation possible for less modest experiment parameters
- Keep initial beam size small small and converging
  - Bring source closer to foils and shape proton emission surface
    Examples: Utsunomiya Univ Research;
    Kawata et al, IEEE Trans. Plasma Sci **36**, 363 (2008)
  - Add focus from magnetic solenoid
- Increase proton current
  - Higher perveance makes pinch stronger if initial beam divergence controlled
- Increase energy if perveance can be maintained
  - Allow more foils with less beam loss/scattering and use of larger spacing for easier fabrication
- Consider variable foil spacing to reduce materials in initial length
  - Closer spacing later should allow small spot when pinched
- Consider combined system with magnetic solenoid
  - Magnetic field penetrates foils and can be superimposed
  - Boost of applied field will help most where beam is large

Conclusions

- Approximate envelope model derived to guide experiments
  - Allows rapid analysis of optimization tradeoffs
  - Results relatively independent of form of radial charge distribution
  - Assumptions reasonable when pulse "long", foils are closely spaced and no neutralization past 1^{st} foil
  - Used to predict max foil spacing possible for net focusing and guide initial beam constraints for workable experiments

- Envelope model applied to proposed GSI Laser Proton Experiment: Parameters can achieve pinch focus if divergence of initial beam limited
  - Scaling clarified suggesting parameters for improved experiments

- Simulations with WARP code having less model assumptions are being carried out to check/optimize further: preliminary results appear consistent with envelope model

- More developed version of the model in preparation for publication and will be applied to explore concept for X-Target igniter pulse